Curriculum Vitae

Personal Information

Family Name: *Velichkov*   First Name: *Bozhidar*   Web Page: `www.velichkov.it`

Current Position

Institution: *Université Grenoble Alpes - **UGA***   Department: *Laboratoire Jean Kuntzmann - **LJK***

Since 1 Sept 2014, I am Assistant Professor - *Maître de Conférences* in Grenoble (France)
in the team *Calculus of Variations, Geometry, Images - **CVGI*** at **LJK**, where I work on
free boundary problems arising in **calculus of variations** and **shape optimization**.

PhD Thesis

Scuola Normale Superiore - **SNS** and Université de Savoie
Discussed on 8 Nov 2013 in SNS (Pisa, Italy). Mention: 70/70 cum Laude.

Title: **Existence and Regularity Result for Some Shape Optimization Problems**
Published in: Edizioni della Normale, Tesi 19, Springer 2015, ISBN 978-88-7642-526-4

Advisors: Giuseppe Buttazzo (Università di Pisa*) and Dorin Bucur (Université de Savoie**)

* As a PhD student at SNS I was free to choose my advisor independently of his affiliation.
** Universitè de Savoie and Université Grenoble Alpes have common PhD program.

Cursus

02/2014 - 08/2014. I spent six months as **Post-doc** in Shape Optimization at Università di Pisa, where
I worked on shape optimization [A12,B2] and optimal transport problems [A15].
(For the references [A12], [B2] *and* [A15], *see the complete list of publications.)*

10/2010 - 11/2013. **PhD** at SNS and Université de Savoie. I won a PhD position at SNS in 2010 and
I chose as advisor G. Buttazzo. In 2011 we started collaborating with D. Bucur; a joint supervision
agreement between SNS and Université de Savoie was signed in March 2012.

10/2005 - 10/2010. I was **Student in Mathematics at SNS** and I graduated with honors: 70/70 cum
Laude, Diploma 1235. As every SNS student, I was also a student at the University of Pisa:
- 2008 - 2010. Master in Mathematics - Università di Pisa (110/110 cum Laude, Diploma 254324)
- 2005 - 2008. Bachelor in Mathematics - Università di Pisa (110/110 cum Laude)

Honors and awards

2017. I was qualified for Associate Professor by MIUR (Italian Ministry of Education and Research).
2015. I won PEDR - *Prime d'Encadrement Doctoral et de Recherche*
2015. I was P.I. of the project *VariForm* financed by Université Grenoble Alpes.
2013. My PhD Thesis was selected for publication in *Edizioni della Normale*.

Links

My personal web page: `www.velichkov.it`
My page on the preprint server CVGMT: `http://cvgmt.sns.it/person/336/`
My ArXiv page: `https://arxiv.org/a/velichkov_b_1.html`
My page at Scopus: `Author ID: 55382990900`
ORCID ID: `https://orcid.org/0000-0003-4968-3087`

Contents

PhD students

Baptiste Trey (09/2016 - 09/2019). Baptiste is a PhD student financed by the collaborative project GeoSpec (2016-2020) between LJK* and IF**. He is co-advised by myself (LJK, 50%) and Emmanuel Russ (IF, 50%). Baptiste is working on the existence and the regularity techniques in shape optimization. Recently, using different techniques from the regularity of the free boundaries, we proved new regularity results for several different classes of shape optimization problems. In

E. Russ, B. Trey, B. Velichkov : *Existence and regularity of optimal shapes for elliptic operators with drift.* Preprint arXiv:1810.07943

we studied the optimal sets for the first eigenvalue of an operator with drift, while in

L. Spolaor, B. Trey, B. Velichkov : *Free boundary regularity for a multiphase shape optimization problem.* Preprint arXiv:1810.06963

we used an epiperimetric inequality approach to conclude the proof of the regularity of the solutions of the multiphase optimization problem studied in

B. Bogosel, B. Velichkov. *Multiphase Optimization Problems for Eigenvalues: Qualitative Properties and Numerical Results.* SIAM J. Numer. Anal. 54 (1) (2015), 210–241.

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*LJK = Laboratoire Jean Kuntzmann - Applied Mathematics Dept, Université Grenoble Alpes.
**IF = Institut Fourier - Mathematics Department, Université Grenoble Alpes.

François Générau (09/2017 - 09/2020). François is a PhD student in my department Laboratoire Jean Kuntzmann, but his thesis is financed by École Normale Supérieure (Paris). He is co-advised by myself (50%) and Édouard Oudet (LJK), who is a specialist in numerical methods for shape optimization problems. During his first year François worked on obstacle-type problems and discovered a new variational approach for the computation of the λ-cut-locus of the distance function to a point on a Riemannian manifold. The method is based on the approximation with solutions of the elastic-plastic torsion problem on a manifold and is inspired by

G. Buttazzo, E. Oudet, B. Velichkov. *A free boundary problem arising in PDE optimization.* Calc. Var. PDE 54 (4) (2015), 3829–3856.

The Hausdorff convergence of the approximating λ-contact sets is a delicate question whose solution requires some fine regularity estimates. The preprint is to appear soon.

COMMISSIONS OF TRUST

I am **Guest Editor** of the special edition "*Free boundary problems and applications*" of the peer-reviewed mathematics journal *Mathematics in Engineering*.

I am **referee** for numerous journals, among which the following:
- ARMA - Archive for Rational Mechanics and Analysis;
- JFA - Journal of Functional Analysis;
- CalcVar - Calculus of Variations and Partial Differential Equations;
- Analysis & PDE;
- ESAIM COCV - Control, Optimisation and Calculus of Variations;
- JMAA - Journal of Mathematical Analysis and Applications;
- JMPA - Journal de Mathématiques Pures et Appliquées;
- SIMA - SIAM Journal on Mathematical Analysis;
- JEMS - Journal of the European Mathematical Society.

2018. I was referee and member of the evaluation committee of the PhD thesis
Shape Optimization Problems for Integral Functionals
and Regularity Properties of Optimal Domain.
Candidate: Harish Shrivastava (Università di Pisa).
Advisor: Giuseppe Buttazzo (Università di Pisa).
Head of the PhD School: Giovanni Alberti (Università di Pisa).
Date of the discussion: 3/12/2018.

2018. I was referee of a project for the call for the award of research grants for young researchers *Progetti Giovani Ricercatori* 2018 of *INdAM - Istituto Nazionale di Alta Matematica "F. Severi".*

2017. I was member of the evaluation committee for the position ATER - *Attaché temporaire d'enseignement et de recherche* (one-year lecturer position) at Laboratoire Jean Kuntzmann (Université Grenoble Alpes) for the academic year 2017-2018. Chair: Eric Blayo.

2016. I was member of the evaluation committee for the position ATER - *Attaché temporaire d'enseignement et de recherche* (one-year lecturer position) at Laboratoire Jean Kuntzmann (Université Grenoble Alpes) for the academic year 2016-2017. Chair: Eric Blayo.

2016. I was member of the evaluation committee (composed by myself, Édouard Oudet and Emmanuel Russ) for the three-years PhD position on the project GeoSpec - *Geometry and Spectral Optimization*.

2015. I was member of the evaluation committee for the position ATER - *Attaché temporaire d'enseignement et de recherche* (one-year lecturer position) at Laboratoire Jean Kuntzmann (Université Grenoble Alpes) for the academic year 2015-2016. Chair: Laurant Desbat.

2013. I was member of the grading committee of the (written) entrance exam at Scuola Normale Superiore. Chair: Stefano Marmi.

Organization of workshops, conferences and seminars

2016. I organized the workshop

Journées ANR SHAPO Shape Optimization

This workshop was also the opening meeting of the project ANR SHAPO; it took place in Grenoble in December 2018 and was co-organized with Charles Dapogny.

2017. I am the organizer of a cycle of workshops dedicated on Calculus of Variations and free Boundary Problems. The first workshop took place in 2017

Calculus of Variations and free Boundary Problems

and was co-organized with Emmanuel Russ and was financed by the project GeoSpec.

2018. The second workshop

Calculus of Variations and free Boundary Problems II

took place in 2018 and was co-organized with Dorin Bucur and was financed by the projects GeoSpec, ANR CoMeDiC and the IUF grant of Dorin Bucur.

2016. I was member of the local organizing committee of the international conference

PICOF - Problèmes Inverses, Contrôle et Optimisation de Formes

that took place in Autrans (Grenoble) in 2016. The other members of the local organizing committee were Charles Dapogny, Eric Bonnetier and Edouard Oudet.

2015. I took part in the organization of the workshop

Calculus of Variations and PDEs

co-organized with Dorin Bucur and Marguerite Gisclon.

Since 2014, I am organizing the seminars

Calcul des Variations, Géométrie Images

of the research groups CVGI and EDP at Laboratoire Jean Kuntzmann (LJK). The seminars take place each Thursday at LJK, and are co-organized with Charles Dapogny (CVGI), Clément Jourdana (EDP) and Ludovic Métivier (EDP).

2014. During my pos-doc at Università di Pisa, I was organizing the seminars

Calculus of Variations and Geometric Measure Theory

together with Agnese Di Castro.

Teaching

Total hours of teaching per year.

2018-2019 (first semester) - 66 hours of teaching
2017-2018 - 163 hours
2016-2017 - 183.5 hours
2015-2016 - 144.5 hours
2014-2015 - 101.5 hours

Teaching and responsibilities, academic year 2018-2019.

• Principal instructor and coordinator (*responsable* (fr.), *titolare del corso* (it.)) of the teaching unit
MAT 102 - Mathématiques outils pour les sciences et l'ingénierie.

— Web page: `http://www.velichkov.it/mat102.html`.
— Degree courses (major, *corso di laurea* (it.), *licence* (fr.)): Chemistry and Biochemistry (Licence Chimie et biochimie (fr.)), Geologia (Licence Sciences de la Terre (fr.)), Engineering Sciences (Licence Sciences pour l'Ingenieur (fr.)).
— Total number of students: 390 divided in 13 groups (*groupes TD* (fr.)) of 30 students each.
— My teaching responsibilities as a coordinator (*responsable d'UE* (fr.)): preparation of the program and the lecture notes; preparation of the exams, both mid-terms (*compitini* (it.), *contrôles continus* (fr.)) and finals; coordination of the teaching instructors (*chargés de TD* (fr.)) - 14 in total.
— My administrative responsibilities as a coordinator (*responsable d'UE* (fr.)): Supervise the preparation of the lecture's schedule; participate to the teaching committees (*jurys* (fr.), *commissioni didattiche* (it.)) of each degree course.

• I was also teaching instructor (*chargé de TD* (fr.)) of the group *CHB-INT*, of the teaching unit *MAT 102*, Degree (major, *laurea* (it.), *licence* (fr.))) in Chemistry and Biochemistry, 32 students, 66 hours of teaching (*76,5 HTD* (fr.)).

Teaching and responsibilities, academic year 2017-2018.

• Principal instructor and coordinator (*responsable* (fr.), *titolare del corso* (it.)) of the teaching unit
MAT 102 - Mathématiques outils pour les sciences et l'ingénierie.

— Web page: `http://www.velichkov.it/mat102.html`.
— Degree courses (major, *corso di laurea* (it.), *licence* (fr.)): Chemistry and Biochemistry (Licence Chimie et biochimie (fr.)), Geologia (Licence Sciences de la Terre (fr.)), Engineering Sciences (Licence Sciences pour l'Ingenieur (fr.)).
— Total number of students: 360 divided in 12 groups (*groupes TD* (fr.)) of 30 students each.
— My teaching responsibilities as a coordinator (*responsable d'UE* (fr.)): see 2018-2019.
— My administrative responsibilities as a coordinator (*responsable d'UE* (fr.)): see 2018-2019.

• I was teaching instructor (*chargé de TD* (fr.)) of the group *SPI-02* , of the teaching unit *MAT 102*, Degree (major, *laurea* (it.), *licence* (fr.))) in Engineering Sciences, 30 students, 66 hours of teaching (*76,5 HTD* (fr.)).

• I was teaching instructor (*chargé de TD* (fr.)) of the group *CeB-03*, of the teaching unit *MAT 102*, Degree (major, *laurea* (it.), *licence* (fr.))) in Chemistry and Biochemistry, 30 students, 66 hours of teaching (*76,5 HTD* (fr.)).

• I was teaching instructor (*chargé de TD* (fr.)) of the group *CeB-05*, of the teaching unit *MAT 102*, Degree (major, *laurea* (it.), *licence* (fr.))) in Chemistry and Biochemistry, 30 students, 33 hours of teaching (*38.25,5 HTD* (fr.)).

Teaching and responsibilities, academic year 2016-2017.

• Principal instructor and coordinator (*responsable* (fr.), *titolare del corso* (it.)) of the teaching unit

MAT 102 - Mathématiques outils pour les sciences et l'ingénierie.

  – Web page: `http://www.velichkov.it/mat102.html`.

  – Degree courses (major, *corso di laurea* (it.), *licence* (fr.)): Chemistry and Biochemistry (Licence Chimie et biochimie (fr.)), Geologia (Licence Sciences de la Terre (fr.)), Engineering Sciences (Licence Sciences pour l'Ingenieur (fr.)).

    – Total number of students: 360 divided in 12 groups (*groupes TD* (fr.)) of 30 students each.

    – My teaching responsibilities as a coordinator (*responsable d'UE* (fr.)): see 2018-2019.

    – My administrative responsibilities as a coordinator (*responsable d'UE* (fr.)): see 2018-2019.

• I was teaching instructor (*chargé de TD* (fr.)) of the group *SPI-02* , of the teaching unit *MAT 102*, Degree (major, *laurea* (it.), *licence* (fr.))) in Engineering Sciences, 30 students, 66 hours of teaching (*76,5 HTD* (fr.)).

• I was teaching instructor (*chargé de TD* (fr.)) of the group *CeB-03* , of the teaching unit *MAT 102*, Degree (major, *laurea* (it.), *licence* (fr.))) in Chemistry and Biochemistry, 30 students, 66 hours of teaching (*76,5 HTD* (fr.)).

• I was also teaching instructor (*chargé de TD* (fr.), *esercitatore* (it.)) of the graduate course

Introduction aux équations différentielles ordinaires et partielles

for the first year of the Master Degree (*Laurea Specialistica* (it.)) in Mathematics at Université Grenoble Alpes, 20 studenti, 45 hours (*45 HTD* (fr.)).

Teaching and responsibilities, academic year 2015-2016.

• Principal instructor and coordinator (*responsable* (fr.), *titolare del corso* (it.)) of the teaching unit

MAT 116 - Mathématiques outils pour les sciences et l'ingénierie.

  – Degree courses (major, *corso di laurea* (it.), *licence* (fr.)): Chemistry and Biochemistry (Licence Chimie et biochimie (fr.)), Geologia (Licence Sciences de la Terre (fr.)), Engineering Sciences (Licence Sciences pour l'Ingenieur (fr.)).

    – Total number of students: 210 divided in 7 groups (*groupes TD* (fr.)) of 30 students each.

    – My teaching responsibilities as a coordinator (*responsable d'UE* (fr.)): see 2018-2019.

    – My administrative responsibilities as a coordinator (*responsable d'UE* (fr.)): see 2018-2019.

• I was teaching instructor (*chargé de TD* (fr.)) of the group *ST-02* , of the teaching unit *MAT 116*, for the first year of the Degree (major, *laurea* (it.), *licence* (fr.)) in Geology, 30 students, 66 hours of teaching (*76,5 HTD* (fr.)).

• I was teaching instructor (*chargé de TD* (fr.)) for one of the groups of the teaching unit

MAT 115 - Algèbre, géométrie et calcul infinitésimal pour la physique

for the first year of the Degree (major, *laurea* (it.), *licence* (fr.)) in Physics at Université Grenoble Alpes, 30 students, 33 hours of teaching (*38,25 heures TD* (fr.)). Coordinator: Claudine Chaffy.

• I was also teaching instructor (*chargé de TD* (fr.), *esercitatore* (it.)) of the graduate course

Introduction aux équations différentielles ordinaires et partielles

for the first year of the Master Degree (*Laurea Specialistica* (it.)) in Mathematics at Université Grenoble Alpes, 20 studenti, 45 hours (*45 HTD* (fr.)). Chief instructor: Eric Dumas.

Teaching and responsibilities, academic year 2014-2015.

• I was teaching instructor (*chargé de TD* (fr.)) for one of the groups of the teaching unit

MAT 115 - Algèbre, géométrie et calcul infinitésimal pour la physique

for the first year of the Degree (major, *laurea* (it.), *licence* (fr.)) in Mathematics at Université Grenoble Alpes, 30 students, 66 hours of teaching (*76,5 heures TD* (fr.)). Coordinator (*responsable UE* (fr.), *titolare del corso* (it.)): Claudine Chaffy.

• I was also teaching instructor (*chargé de TD* (fr.), *esercitatore* (it.)) of the graduate course

Introduction aux équations différentielles ordinaires et partielles

for the first year of the Master Degree (*Laurea Specialistica* (it.)) in Mathematics at Université Grenoble Alpes, 20 studenti, 45 hours (*45 HTD* (fr.)). Chief instructor (*titolare del corso* (it.)): Eric Dumas.

Teaching during my PhD studies in Pisa.

• **2012-2013.** I was teaching instructor (*esercitatore* (it.), *chargé de TD* (fr.)) for the course

Analisi Matematica I

of the first year of the Degree course (*corso di laurea* (it.), *licence* (fr.)) in Mechanical Engineering of Università di Pisa, 100-150 students, 50 hours of teaching. Chief instructor (*Titolare del corso* (it.)): Giuseppe Buttazzo.

• **2011-2012.** I was teaching instructor (*esercitatore* (it.), *chargé de TD* (fr.)) for the course

Analisi Matematica I

of the first year of the Degree course (*corso di laurea* (it.), *licence* (fr.)) in Mechanical Engineering of Università di Pisa, 100-150 students, 50 hours of teaching. Chief instructor (*Titolare del corso* (it.)): Giuseppe Buttazzo.

• **2011-2012.** I was teaching instructor (*esercitatore* (it.), *chargé de TD* (fr.)) for the course *Analisi Matematica - corso di recupero*, supplementary course for the Bachelor Degree (. *Laurea Triennale* (it.)) in Mathematics at Università di Pisa, 20 students, 20 hours of teaching.

CONFERENCES AND SEMINARS

Invited talks at conferences and workshops

Recent results on the regularity of the free boundary of the obstacle problem.
Calculus of Variations at Paris-Diderot (Paris, 2018). Link (to the site of the conference).

Shape optimization problems for elliptic operators with drift.
Journées Optimisation de Formes et Applications 2 (Pau, 2018). Link.

On the honeycomb conjecture for a class of minimal convex partitions.
Transport problems in Zürich (Zürich, 2017). Link.

An epiperimetric inequality approach to the regularity of the free boundaries.
GMT Shape Optimization and Free Boundaries (SISSA, Trieste, 2016). Link.

Lipschitz regularity for quasi-minimizers and applications to some shape optimization problems.
Calculus of Variations, Geometric Measure Theory, Optimal Transportation: from Theory to Applications (Lyon, 2016). Link.

Regularity of optimal sets for spectral functionals.
Advances in Nonlinear Problems from Material Science and Shape Optimization (Pisa, 2016). Link.

Regularity of optimal sets for spectral functionals.
Bru-To: Bruxelles-Torino PDE's Conference (Torino, 2016). Link.

Reinforcement of an elastic membrane, infinity laplacian and obstacle problems.
Spectral Theory and Shape Optimization Problems for Elliptic PDEs (Milano, 2015). Link.

Tre problemi di ottimizzazione legati al problema dell'ostacolo
Proprietà Analitico-Geometriche di Soluzioni di EDP (Napoli, 2016). Link.

Lipschitz continuity of the eigenfunctions on optimal sets.
Journées EDP Rhone-Alpes-Auvergne 2014 (Ecully, 2014). Link.

Short talks (20-30 min) at conferences and workshops

Variational approach to the regularity of optimal sets
VII PDE, Optimal Design and Numerics (Benasque, 2017). Link.

An epiperimetric inequality approach to the regularity of the free boundaries.
Shape Optimization and Isoperimetric and Functional Inequalities (CIRM Luminy, 2016). Link.

Regularity of the optimal sets for spectral functionals.
XIII Colloque Franco-Roumain de Mathématiques Appliquées (Iaşi, 2016). Link.

A free boundary problem arising in PDE optimization.
XXVI Convegno Nazionale di Calcolo delle Variazioni (Levico Terme, 2016). Link.

A free boundary problem arising in PDE optimization.
Shape Optimization and Spectral Geometry (ICMS Edinburgh, 2015). Link.

Lipschitz continuity of the eigenfunctions on optimal sets.
Workshop Shape and topological optimization (RICAM, Linz, 2014). Link.

Shape optimization problems for spectral functionals with perimeter and volume terms.
XXVI Convegno Nazionale di Calcolo delle Variazioni (Levico Terme, 2014). Link.

Multiphase shape optimization problems.
New Trends in Shape Optimization (Erlangen, 2013). Link.

The spectral drop problem.
Partial Differential Equations, Optimal Design and Numerics (Benasque, 2013). Link.

Shape optimization problems with internal constraint.
Shape Optimization Problems and Spectral Theory (CIRM Luminy, 2012). Link.

Shape optimization problems with internal constraint.
XXVI Convegno Nazionale di Calcolo delle Variazioni (Levico Terme, 2012). Link.

Shape Optimization Problems on Metric Measure Spaces.
Geometric Analysis of Sub-Riemannian and Metric Spaces (Pisa, 2011). Link.

Seminars

Regularity of the free boundaries around isolated singularities.
Università degli studi di Napoli Federico II, 26/03/2018.

Variational approach to the regularity of the singular free boundaries.
ETH Zürich,13/03/2018.

Regularity of the free boundaries around isolated singularities.
Calcolo delle Variazioni ed Analisi Geometrica, Università di Pisa, 07/02/2018.

Regularity of the free boundaries around isolated singularities.
Université Paris Diderot, 05/02/2018.

Regularity of the free boundaries around isolated singularities.
Université Paris Sud, 26/01/2018.

Variational approach to the regularity of the free boundaries.
Université Aix-Marseille, 26/10/2017.

Regularity of the optimal sets for spectral functionals.
Max Planck Institut Leipzig, 13/05/2016.

Regularity of the optimal sets for spectral functionals.
Unuversität Zürich, 13/04/2016.

Regularity of the optimal sets for spectral functionals.
Politecnico di Milano, 20/04/2016.

Lipschitz continuity of the eigenfunctions on optimal sets.
Università degli Studi di Torino, 29/10/2014.

Régularité Lipschitz pour les fonctions propres sur ensembles optimaux.
Séminaire Parisien d'Optimisation, 6/10/2014.

Optimal domains for spectral functionals with perimeter and volume penalizations.
Séminaires Analyse Appliquée LATP Marseille, 08/04/2014.

Optimal domains for spectral functionals with perimeter and volume penalizations.
Séminaires LJK - Géométrie - Images - Calcul des Variations, Université de Grenoble, 02/04/2014.

Existence and regularity for some spectral optimization problems with perimeter constraint.
Université de Savoie, 09/11/2012.

Subsolutions of the Dirichlet energy functional. Density estimates and applications.
Università di Pavia, 08/05/2012.

Concentration-compactness principle and shape optimization problems.
Calcolo delle Variazioni ed Analisi Geometrica, Università di Pisa, 25/05/2011.

RESEARCH PERIODS AND INVITATIONS

- 21/10/2018 - 02/11/2018. SISSA Trieste. Invited by Guido De Philippis.

- 27-31/03/2018. SISSA Trieste. Invited by Guido De Philippis.

- 05-17/03/2018. ETH Zürich. Invited by Maria Colombo.

- 20/02/2018-03/03/2018. Politecnico di Milano. Invited by Ilaria Fragalà.

- 05-11/02/2018. Università di Pisa. Invited by Giuseppe Butazzo.

- 20-24/03/2017. Politecnico di Milano. Invited by Ilaria Fragalà.

- 08-31/01/2017. Università di Pisa. Invited by Giuseppe Butazzo.

- 06-10/06/2016. SISSA Trieste. Invited by Giovanni Franzina.

- 09-13/05/2016. Max Planck Institut Leipzig. Invited by Luca Spolaor.

- 18-22/03/2016. Politecnico di Milano. Invited by Ilaria Fragalà.

- 11-15/04/2016. Universität Zürich. Invited by Annalisa Massaccesi.

- 04-08/04/2016. Università degli Studi di Torino. Invited by Susanna Terracini.

- 03-31/01/2016. Università di Pisa. Invited by Giuseppe Buttazzo.

- 13-17/07/2015. Università degli Studi di Torino. Invited by Susanna Terracini.

- 02-06/03/2015. Università di Pisa. Invited by Giuseppe Buttazzo.

- 26-30/10/2014. Università degli Studi di Torino. Invited by Susanna Terracini.

- 09-18/04/2014. Isaac Newton Institute, Cambridge. Visit during the research period
Free boundary problems and related topics.

- 19-21/03/2014. Université Paris Dauphine. Invited by Jimmy Lamboley.

- 07-11/04/2013. Friedrich Alexander Universität Erlangen - Nürnberg. Invited by Aldo Pratelli.

- 06-08/05/2012. Università di Pavia. Invited by Aldo Pratelli.

LIST OF PUBLICATIONS

Preprints

[P5] E. Russ, B. Trey, B. Velichkov : *Existence and regularity of optimal shapes for elliptic operators with drift.* Preprint arXiv:1810.07943

[P4] L. Spolaor, B. Trey, B. Velichkov : *Free boundary regularity for a multiphase shape optimization problem.* Preprint arXiv:1810.06963

[P3] M. Colombo, L. Spolaor, B. Velichkov : *On the asymptotic behavior of the solutions to parabolic variational inequalities.* Preprint arXiv:1809.06075

[P2] D. Mazzoleni, S. Terracini, B. Velichkov : *Regularity of the free boundary for the vectorial Bernoulli problem.* Preprint arXiv:1804.09243

[P1] M. Engelstein, L. Spolaor, B. Velichkov : *Uniqueness of the blow-up at isolated singularities for the Alt-Caffarelli functional.* Preprint arXiv:1801.09276

Journal articles

[A27] M. Colombo, L. Spolaor, B. Velichkov : *Direct epiperimetric inequalities for the thin obstacle problem and applications.* Accepted: **Comm. Pure. Appl. Math.** (Preprint arXiv:1709.03120)

[A26] M. Engelstein, L. Spolaor, B. Velichkov : *(Log-)epiperimetric inequality and regularity over smooth cones for almost area-minimizing currents.* **Geometry & Topology** (2018). https://msp.org/scripts/coming.php?jpath=gt

[A25] I. Fragalà, B. Velichkov : *Serrin-type theorems for triangles.* **Proc. Amer. Math. Soc.** (2018). https://doi.org/10.1090/proc/14352

[A24] G. Buttazzo, F. Maestre, B. Velichkov : *Optimal potentials for problems with changing sing data.* **J. Optim. Theory Appl.** 178 (3) (2018), 742–762.

[A23] M. Colombo, L. Spolaor, B. Velichkov : *A logarithmic epiperimetric inequality for the obstacle problem.* **Geom. Funct. Anal.** 28 (4) (2018), 1029–1061.

[A22] G. Buttazzo, B. Velichkov : *A shape optimal control problem with changing sign data.* **SIAM J. Math. Anal.** 50 (3) (2018), 2608–2627.

[A21] L. Spolaor, B. Velichkov : *An epiperimetric inequality for the regularity of some free boundary problems: the 2-dimensional case.* **Comm. Pure. Appl. Math.** 72 (2) (2018), 375–421.

[A20] G. De Philippis, J. Lamboley, M. Pierre, B. Velichkov : *Regularity of minimizers of shape optimization problems involving perimeter.* **J. Math. Pure. Appl.** 109 (2018), 147–181.

[A19] D. Bucur, I. Fragalà, B. Velichkov, G. Verzini : *On the honeycomb conjecture for a class of minimal convex partitions.* **Trans. Amer. Math. Soc.** 370 (10) (2018), 7149–7179.

[A18] D. Mazzoleni, S. Terracini, B. Velichkov : *Regularity of the optimal sets for some spectral functionals.* **Geom. Funct. Anal.** 27 (2017), 373–426.

[A17] A. Massaccesi, E. Oudet, B. Velichkov : *Numerical Calibration of Steiner trees.* **Appl. Math. Optim.** (2017). https://doi.org/10.1007/s00245-017-9421-5

[A16] J.C. Bellido, G. Buttazzo, B. Velichkov : *Worst-case shape optimization for the Dirichlet energy.* **Nonlinear Analysis** 153 (2017), 117–129.

[A15] G. De Philippis, A. Meszaros, F. Santambrogio, B. Velichkov : *BV Estimates in Optimal Transportation and Applications.* **Arch. Rat. Mech. Anal.** 219 (2) (2016), 829–860.

[A14] D. Bucur, D. Mazzoleni, A. Pratelli, B. Velichkov : *Lipschitz Regularity of the Eigenfunctions on Optimal Domains.* **Arch. Rat. Mech. Anal.** 216 (2015), 117–151.

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[A8] G. Buttazzo, B. Ruffini, B. Velichkov : *Spectral optimization problems for metric graphs.* **ESAIM: COCV** 20 (1) (2014) 1–22.

[A7] B. Velichkov : *A Note on the Monotonicity Formula of Caffarelli-Jerison-Kenig.* **Rend. Lincei Mat. Appl.** 25 (2014), 165–189.

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[A5] D. Bucur, G. Buttazzo, B. Velichkov : *Spectral Optimization Problems for Potentials and Measures.* **SIAM J. Math. Anal.** 46 (4) (2014), 2956–2986.

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[A1] G. Buttazzo, B. Velichkov : *Shape optimization problems on metric measure spaces.* **J. Funct. Anal.** 264 (1) (2013), 1–33.

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[B4] G. Buttazzo, B. Velichkov : *Spectral optimization problems for Schrödinger operators.* Shape Optimization and Spectral Theory, De Gruyter (2017), 325–352.

[B3] G. Buttazzo, B. Velichkov : *The spectral drop problem.* Contemporary Mathematics **666** (2016), 111–135.

[B2] G. Buttazzo, M. Van Den Berg, B. Velichkov : *Optimization problems involving the first Dirichlet eigenvalue and the torsional rigidity.* New Trends in Shape Optimization, Springer (2015), 19–41.

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Monographs

[M1] B. Velichkov : *Existence and regularity results for some shape optimization problems.* Edizioni della Normale, Tesi 19, Springer 2015, ISBN 978-88-7642-526-4.

FUNDING ID

Project Title	Funding source	Amount	Period (mm/yyyy)	My role
Shape Optimization - **ShapO**	ANR	300kE	10/2018 - 10/2022	local coordinator
Convergent Metrics for Digital Calculus - **CoMeDiC**	ANR	400kE	10/2015 - 09/2020	member
Geometry and Spectral Optimization - **GeoSpec**	ANR	250kE	09/2016 - 02/2020	member
Méthodes Variationnelles en Optimisation de Formes - **VariForm**	UGA	15kE	4/2015 - 12/2016	P.I.

ANR = Agence Nationale de la Recherche   UGA = Université Grenoble Alpes

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I was the **Principal Inverstigator** of the project
VariForm - "Méthodes Variationnelles en Optimisation de Formes"
financed by the program "Alpes Grenoble Innovation Recherche" of Université Grenoble Alpes.

Duration: 2 years, 2015-2016. **Volume**: 15kE.

Summary: This project was dedicated to the study of monotonicity formulas and their applications to the regularity of the free boundaries arising in variational free boundary and shape optimization problems. My main contributions, developed in the context of this project are the following:

[MTV] D. Mazzoleni, S. Terracini, B. Velichkov. *Regularity of the optimal sets for some spectral functionals.* **Geom. Funct. Anal.** 27 (2017), 373–426.

[SV] L. Spolaor, B. Velichkov. *An epiperimetric inequality for the regularity of some free boundary problems: the* 2-*dimensional case.* **Comm. Pure. Appl. Math.** (2018).
`https://doi.org/10.1002/cpa.21785`

The papers [MTV] and [SV] were fundamental for my subsequent research on the regularity of the free boundaries. The methods pioneered in [SV] later led to the introduction of the *logarithmic epiperimetric inequality*.

I am **local coordinator of the national project**

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ANR ShapO - "Shape Optimization"

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financed by ANR - Agence Nationale de Recherche.

Coordinator: Jimmy Lamboley.
Local coordinators: Ilaria Lucardesi, Berardo Ruffini, Bozhidar Velichkov.
Volume: 300kE. **Duration**: 4 years (2018-2022).

Summary: This project has 22 members (and 10 non-permanent members: PhD students and post-docs). It covers a large spectrum of shape optimization problems and gathers a big part of the shape optimization community in France.

Administrative organization: The projects involves 4 partners. I am local coordinator of "Partner 2 - Grenoble" with 5 permanent members (Edouard Oudet, Emmanuel Russ, Charles Dapogny, Dorin Bucur) and 4 PhD students.

Scientific organization (research groups): The members of the project are organized in the following research groups (teams), working on different parts (tasks) of the project:
1.1. *Principal eigenvalue for non-symmetric/non-local elliptic operators.*
1.2. *Charged liquid drops.* **1.3.** *Uncertainties.*
2.1. *Topological constraints.* **2.2.** *Global geometrical constraints.*
2.3. *Partitions and optimization among polygons.* **2.4.** *Engineering constraints.*
3.1. *Shape flow with respect to the first eigenvalue of the Dirichlet Laplacian.*
3.2. *Shape flows for general cost functionals.* **3.3.** *Evolution problems for potentials and measures.*
4.1. *Numerical investigations around geometric constraints.*
4.2. *Modeling and numerical implementation for problems issued from mechanical engineering, physics and life sciences.*
I am scientific coordinator of the research group **3.1** and member of **1.3**, **2.1**, **2.3** and **3.3**.
I am also organized the first meeting of the project, which will be in Grenoble, 6-7/12/2018.

I am **scientific coordinator** of the **research group**

<div align="center">

"*Shape flow with respect to the first eigenvalue of the Dirichlet Laplacian*"

</div>

of the national project ANR ShapO "Shape Optimization".

Permanent members:

- Bozhidar Velichkov (Maître de Conférences, Université Grenoble Alpes, coordinator);
- Virginie Bonnaillie-Noël (Directeur de Recherche, CNRS);
- Dorin Bucur (Full Professor, Université de Savoie);
- Gisella Croce (Maître de Conférences, Université du Havre);
- Marc Dambrine (Full Professor, Université de Pau);
- Jimmy Lamboley (Full Professor, Université Pierre et Marie Curie).

Summary of the objectives: Shape evolution problems have a big impact on applied sciences, and they also have important applications to questions with purely theoretical taste as for example, the regularity of free boundary problems. Of particular interest is the shape flow on the sphere defined with respect to the functional $\mathcal{F}(\Omega) := \lambda_1(\Omega) + Vol(\Omega)$, where $\lambda_1(\Omega)$ is the first eigenfunction of the spherical domain Ω. The construction of such a flow is one of the main objectives of ShapO.

My role in the research group:
- I participate as a scientific coordinator and an expert in the regularity of the free boundaries.
- I defined the main objectives of the research group and wrote the related part of the project proposal.

I am **member** of the **research group**

"*Uncertainties*"

of the project ANR ShapO "Shape Optimization".

Permanent members:

- Marc Dambrine (Full Professor, Université de Pau, coordinator);
- Grégoire Allaire (Full Professor, École Polytechnique);
- Charles Dapogny (Researcher CNRS);
- Bozhidar Velichkov (Maître de Conférences, Université Grenoble Alpes).

Summary of the objectives: We aim to model new types of uncertainties arising in shape optimization, for instance, those incurred by the manufacturing processes of mechanical or physical devices. We aim to develop stochastic calculus for randomly perturbed shapes: given a probability space of random perturbations acting on a given shape, we want to study tools to compute the mean and the variance of a given shape functional. The existence of optimal domains with respect to this type of stochastic shape functionals is a challenging open question: the functionals obtained through this kind of procedure are typically not monotone and the usual existence theorems cannot be applied, so we aim to develop new techniques to cover this case.

My role in the research group:

- I participate as an expert in the theoretical aspects of shape optimization problems involving uncertainties.

I am **member** of the **research group**

"*Topological constraints*"

of the national project ANR ShapO "Shape Optimization".

Permanent members:

- Antoine Lemenant (Maître de Conférences, Université Paris Sud, coordinator);
- Dorin Bucur (Full Professor, Université de Savoie);
- Antoine Henrot (Full Professor, École des Mines de Nancy);
- Jimmy Lamboley (Full Professor, Université Pierre et Marie Curie),
- Bozhidar Velichkov (Maître de Conférences, Université Grenoble Alpes).

Summary of the objectives: The aim of this research group is to study shape optimization problems under topological constraints. There are two main research directions:

(1) *Shape optimization problems for simply connected open sets.* This topological constraint is stable for deformation by vector fields so, at least formally, the optimality conditions lead to overdetermined free boundary problem. However, many of the known regularity techniques are hard to apply, since the constraint is of non-local nature. This is related, for instance, to the isoperimetric conjecture on the buckling load of the clamped plate (Polya and Szego, 1954).

(2) *Shape optimization problems involving closed connected sets of dimension one.* A large class of free discontinuity problems involves one dimensional connected sets, the objective being to minimize an energy associating a physical energy and the length. An important example is the compliance problem from mechanics, where one considers a membrane subject to some deformation, which is attached to a certain unknown connected one-dimensional Dirichlet region Σ. One of our main objectives is to study the regularity of the optimal sets Σ arising in this context.

My role in this research group:

- I participate as an expert in the regularity of the free boundaries and the optimal shapes.

I am **member** of the **research group**

"*Partitions and optimization among polygons*"

of the national project ANR ShapO "Shape Optimization".

Permanent members:

- Dorin Bucur (Full Professor, Université de Savoie, coordinator);
- Virginie Bonnaillie-Noel (Directeur de Recherche, CNRS);
- Jimmy Lamboley (Full Professor, Université Pierre et Marie Curie);
- Enea Parini (Maître de Conférences, Université Aix-Marseille);
- Yannick Privat (Full Professor, Université de Strasbourg);
- Bozhidar Velichkov (Maître de Conférences, Université Grenoble Alpes);
- Alessandro Zilio (Maître de Conférences, Université Paris Diderot).

Summary: In 2005-2007, Burdzy, Caffarelli-Lin and Van den Berg conjectured in different contexts that the asymptotic behavior of optimal partitions of a plane domain in cells minimizing the sum of their fundamental Dirichlet Laplacian eigenvalues is given by a honeycomb structure. The interest for this question arises in the study of segregated configurations of competition-diffusion systems, energy minimizing configurations of Bose-Einstein condensates and the structure of the nodal domains of higher order eigenvalues. Recent results, in which members of the group have been involved (Bucur and Velichkov), opened new avenues and reduced the conjecture to understanding the discrete Faber-Krahn inequality on polygons with 5, 6 and 7 edges.

My role in the research group:

• I participate as an expert in the regularity of the optimal shapes for multiphase and optimal partition problems. I will also contribute with my experience in the study of the asymptotic behavior of the optimal partitions.

I am **member** of the **research group**

"*Evolution problems for potentials and measures*"

of the national project ANR ShapO "Shape Optimization".

Permanent members:

- Berardo Ruffini (Maître de Conférences, Université de Montpelier, coordinator);
- Dorin Bucur (Full Professor, Université de Savoie);
- Michael Goldman (Researcher CNRS);
- Ilaria Lucardesi (Maître de Conférences, École des Mines de Nancy);
- Bozhidar Velichkov (Maître de Conférences, Université Grenoble Alpes);
- Alessandro Zilio (Maître de Conférences, Université Paris Diderot).

Summary: We consider variational evolution with respect to cost functionals depending on the solutions of a PDE ruled by Schrödinger potentials $-\Delta + V$ on $\mathbb{R}^d$. In this context, regularity issues are much easier to handle, and since the optimal potentials may approximate the optimal sets, this provides a tool for the numerical approximation of shape flows. As a first test, we will study the functional $V \mapsto \lambda_1(-\Delta + V) + \int F(V(x)) \, dx$. The main objectives are:

• Prove the existence of the flow starting from a non-negative measurable potential.

• Prove that there is a sequence of weight functions F_n, for which the respective flows of potentials converge to a shape flow.

My role in this research group:

• I participate as an expert in spectral optimization problems involving potential and measures.

• I participated in the definition of the main objectives of this research group and I wrote the related part of the project proposal ShapO.

I am member of the **research group**

"*Spectral theory: Vector fields, complexity and diffusion*"

of the project **GeoSpec** "Geometry and Spectral Optimization".

Duration of the project: 42 months (2016-2020), $\sim$ 250kE.

Coordinators: Gérard Besson, Édouard Oudet.

Web page: `https://ljk.imag.fr/GeoSpec/`

Members of the research group:
- Gérard Besson (Directeur de Recherche CNRS);
- Dorin Bucur (Full Professor, Université de Savoie);
- Emmanuel Russ (Full Professor, Université Grenoble Alpes);
- Bozhidar Velichkov (Maître de Conférences, Université Grenoble Alpes);
- my PhD students Baptiste Trey and François Generau.

Summary of the objectives of GeoSpec: The general goal of this project is to combine the knowledge of theoretical and applied mathematicians in order to study optimal manifolds with respect to complex criteria, under complex geometrical constraints. This project aims to develop new approaches in the numerical approximation of optimal or critical manifolds and the analysis of shape optimization problems under complex geometrical constraints. The focus is on problematics in which both theoretical and applied mathematicians are involved. By this proposal, our team targets progress on understanding deeper the following three different fields of metric structures, dynamical systems and spectral theory.

Summary of the objectives of the research groups: We investigate different variational shape optimization and free boundary methods that can be used to approach problems in applied geometry, as for instance, the numerical computation of the cut-locus of a distance function and the closed geodesics of minimal length on a given compact surface.

My role in the project and the research group:
- I participate in GeoSpec as an expert in the regularity for shape optimization problems.
- I participated in defining the objectives of the research group and in writing the project proposal.
- I am advisor of the PhD student Baptiste Trey, whose thesis is financed by GeoSpec.
- I organized the following workshops on thematics related to the project:

> Calculus of Variations and Free Boundary Problems I - Grenoble, 2017.
>   `http://www.velichkov.it/workshop_geospec.html`

> Calculus of Variations and Free Boundary Problems II - Grenoble, 2018.
>   `http://www.velichkov.it/workshop_cvfbp_2.html`

- In march 2017, I gave a talk, on the project's seminar/working group, with the following title:

  Régularité de la frontière libre pour le problème de Bernoulli: une approche variationnelle.

- My main contributions to the project are the following:

> [RTV] E. Russ, B. Trey, B. Velichkov. *Existence and regularity of optimal shapes for elliptic operators with drift.* Preprint arXiv:1810.07943

> [STV] L. Spolaor, B. Trey, B. Velichkov. *Free boundary regularity for a multiphase shape optimization problem.* Preprint arXiv:1810.06963

I am **member** of the national project

ANR CoMeDiC "Convergent Metrics for Digital Calculus"

Duration: 5 years (2015-2020). **Volume**: ∼400kE.

Coordinator: Jacques-Olivier Lachaud.

Web page: `https://lama.univ-savoie.fr/comedic/`

Summary: The project involves 4 teams and 23 members, most of them specialists in Numerical Analysis and Informatics; it aims to develop discrete calculus methods in digitalized spaces. I am part of the team at Laboratoire Jean Kuntzmann (members: me, Edouard Oudet and Boris Thibert) and I am involved in the following work groups:

- Research group on *Subtask 1.2. Adaptation of variational problems to digital calculus*
  - *Coordinator:* Édouard Oudet;
  - *Members:* Dorin Bucur, David Coeurjolly, Jacques-Olivier Lachaud, Laurent Najman, Edouard Oudet, Tristan Roussillon, Hugues Talbot, Bozhidar Velichkov;
  - *Short description of the objectives:* Introduce efficient and relevant discretization of differential operators on singular objects. Express problems involving functionals with discontinuities (like Mumford-Shah and Ambrosio-Tortorelli) or free boundaries in digital calculus. Combine Γ-convergence theory with geometric measure tools to get stronger formulation of convergence for digital calculus. Incorporate known geometric Γ-convergence results into digital calculus.
- Research group on *Subtask 2.3. Digital calculus for shape optimization*
  - *Coordinator:* Édouard Oudet;
  - *Members:* Dorin Bucur, Jacques-Olivier Lachaud, Laurent Najman, Edouard Oudet, Pascal Romon, Hugues Talbot, Boris Thibert, Bozhidar Velichkov;
  - *Short description of the objectives:* Consider digital calculus to approximate optimal geometrical structure of shape optimization problems. Impact on the identification of calibrations to characterize global optimal structures (minimal surfaces, Steiner tree problem,...).

My role and contributions to CoMeDiC:
- Together with Boris Thibert, I organized the project workshop-meeting

  MEETING T24 - LJK (Laboratoire Jean Kuntzmann, Grenoble, 15-16 January 2018).
- In 2017, I gave a talk entitled

  Honeycomb conjectures on convex sets.

on the project meeting-workshop

MEETING T18 - LAMA (Autrans, 12-16 June 2017).
- My main scientific contributions related to the projects are:

  [BFVV] D. Bucur, I. Fragalà, B. Velichkov, G. Verzini.
  On the honeycomb conjecture for a class of minimal convex partitions.
  Trans. Amer. Math. Soc. **370** (10) (2018), 7149–7179.

  [MOV] A. Massaccesi, E. Oudet, B. Velichkov.
  Numerical Calibration of Steiner trees. Appl. Math. Optim. (2017).
  `https://doi.org/10.1007/s00245-017-9421-5`

In the period 2011-2014, as a PhD student, I was an active participant in the research groups of the following (french) national projects:

- ANR Optiform - *Shape optimization*.
  Period: 2012-2016. **Coordinator:** *Antoine Henrot*.
  Web page: `https://optiform.ec-lyon.fr/index.php`

- ANR GAOS - *Geometric analysis of optimal shapes*.
  Period: 2009-2012. **Coordinator:** *Dorin Bucur*.
  Web page: `https://www.lama.univ-savoie.fr/pagesmembres/bucur/WWW-ANR/index.php`

Since 2011, I participated to all the meetings of these projects and I gave the following talks:

- *Shape optimization problems with Neumann boundary conditions*.
  Project meeting ANR Optiform (Nancy, 25/02/2014).

- *Multiphase shape optimization problems*.
  Project meeting ANR Optiform (Paris, 26/11/2013).

- *Existence and regularity for some spectral optimization problems with perimeter constraint*.
  Project meeting ANR Optiform (Rennes, 22/01/2013).

- *Shape optimization problems with internal constraint*.
  Project meeting ANR GAOS (Chambery, 02/06/2011).