Book of abstracts and timetable

Workshop GB70

50 anni di Calcolo delle Variazioni

20 – 22 Maggio 2024

This workshop is organized with the support of the following projects:

- Università di Pisa – "Bando per la concessione di contributi per convegni scientifici e la pubblicazione dei relativi atti (gennaio – giugno 2024)"
- Dipartimento di Matematica – "MIUR Excellence Department Project awarded to the Department of Mathematics, University of Pisa, CUP I57G22000700001"
- MUR-PRIN project 2022R537CS "NO^3 – Nodal optimization, nonlinear elliptic equations, nonlocal geometric problems, with a focus on regularity", granted by the European Union – Next Generation EU;
- ERC project "VAREG – Variational approach to the regularity of the free boundaries" (grant argeement n.853404), granted by the European Research Council (ERC) within the European Union's Horizon 2020 research and innovation programme.

SPEAKERS

GB70 – 50 anni di Calcolo delle Variazioni

20– 22 Maggio 2024

Alessio Brancolini (Modena)
Lorenzo Brasco (Ferrara)
Ariela Briani (Tours, Francia)
Luca Briani (Munich, Germania)
Andrea Davini (Roma)
Luigi De Pascale (Firenze)
Ilaria Fragalà (Milano)
Lorenzo Freddi (Udine)
Augusto Gerolin (Otawa, Canada)
Michele Gori (Firenze)
Luca Granieri (Bari)
Serena Guarino Lo Bianco (Modena - Reggio Emilia)
Xin Yang Lu (Lakehead, Canada)
Francesco Paolo Maiale (L'Aquila)
Francesca Prinari (Pisa)
Berardo Ruffini (Bologna)
Filippo Santambrogio (Lyon, Francia)
Paola Trebeschi (Brescia)
Bozhidar Velichkov (Pisa)

TIMETABLE

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20– 22 Maggio 2024

The workshop will take place in

Aula Magna of Dipartimento di Matematica, Università di Pisa

Lunedì 20/5/2024

09:30–10:00 Registration
10:00–10:40 Freddi
10:40–11:10 Coffee break
11:10–11:40 Faina*, Al-hassem*
11:40–12:20 Davini
12:20–13:00 Briani A
13:00–15:00 Lunch break
15:00–15:40 Briani L
15:40–16:20 Prinari
16:20–16:50 Coffee break
16:50–17:30 Lu
17:30–18:10 Gerolin

Martedì 21/5/2024

09:30–10:10 Brancolini
10:10–10:50 Fragalà
10:50–11:20 Coffee break
11:20–11:40 Belloni*
11:40–12:20 Granieri
12:20–13:00 Gori
13:00–15:00 Lunch break
15:00–15:40 Guarino
15:40–16:20 De Pascale
16:20–16:50 Coffee break
16:50–17:30 Maiale
17:30–18:10 Brasco

Mercoledì 22/5/2024

09:30–10:10 Ruffini
10:10–10:50 Santambrogio
10:50–11:20 Coffee break
11:20–11:40 Shrivastava*
11:40–12:20 Trebeschi
12:20–13:00 Velichkov
13:00–13:30 Closure

*presentazione fatta da Giuseppe Buttazzo.

ABSTRACTS

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Alcuni problemi di matematica del mondo assicurativo

Alessio Brancolini

Abstract: Il seminario si concentrerà sugli aspetti matematici della direttiva Solvency II, il quadro normativo europeo per le compagnie di assicurazione e riassicurazione. Solvency II introduce un approccio quantitativo alla gestione del rischio, richiedendo alle compagnie di assicurazione di calcolare i requisiti di capitale basati sul profilo di rischio. Esploreremo alcuni aspetti matematici di questo approccio.

Buttazzo incontra Hardy

Lorenzo Brasco

Abstract: Discutiamo una disuguaglianza funzionale, nel segno di un celebre risultato di G. H. Hardy.

From the Hamilton-Jacobi equation and control problems with discontinuities to European project management and back again

Ariela Briani

Abstract: I will discuss the notion of viscosity solution for first order Hamilton-Jacobi equations with codimension-1 discontinuities. These kind of problems are intimately related to HJ equations on networks. This can be easy understood looking at an HJ equation on the real line with only one discontinuity at x=0, as a network with two branches. Starting from the 2010s, various group of people investigated this kind of problem for convex Hamiltonians, aiming to give a definition of viscosity solution able to identify the value function of a corresponding optimal control problem, as in the classical framework. I will describe the different definitions mainly with the aim of highlighting the link with the set of admissible trajectories to be considered in the corresponding optimal control problem. My main objective will be to explain how the optimal control point of view is, so far, essential to obtain comparison results and to show where purely PDEs approaches seems insufficient. I will end with some work in progress ans open problems.

Based on my experience as project coordinator for the MSCA Cofund projects, I will also share a few comments, thoughts and suggestions on writing and managing European proposals and programs.

A shape optimization problem involving the torsion function and a related hexagonal structure

Luca Briani

Abstract: Given a domain $\Omega \subset \mathbb{R}^d$ with finite measure the torsion function of Ω is defined to be the unique solution $w_{p,\Omega}$ to the following boundary value problem:

$$-\Delta_p w = 1, \text{ in } \Omega, \quad w = 0 \text{ on } \partial\Omega.$$

In this seminar I will discuss the optimization problems for the *mean-to-max* shape functional

$$\Omega \mapsto \|w_{p,\Omega}\|^{-1}_{L^\infty(\Omega)} \frac{\int_\Omega w_{p,\Omega}(x)dx}{|\Omega|},$$

also known as *efficiency* of the torsion function. Finding optimal bounds to the functional above is related to the problem of comparing the torsional rigidity $T_p(\Omega)$ with the first eigenvalue $\lambda_p(\Omega)$ of the $p-$Laplace operator.

Based on joint works in collaboration with D. Bucur, G. Buttazzo and F. Prinari.

Stochastic Homogenization of viscous HJ equations in 1d

Andrea Davini

Abstract:In this talk we will present some new results we have recently obtained about homogenization of viscous Hamilton-Jacobi equations in dimension one in stationary ergodic environments with nonconvex Hamiltonians. In the non-degenerate case, i.e., when the diffusion coefficient is strictly positive, homogenization is established for superlinear Hamiltonians of fairly general type. When, on the other hand, the diffusion coefficient degenerates, meaning that it is zero at some points or on some regions of the real line, homogenization is proved for Hamiltonians that are additionally assumed quasiconvex in the momentum variable. Furthermore, the effective Hamiltonian is shown to be quasiconvex. This latter result is new even in the periodic setting, despite homogenization has been known for quite some time.

A minimalist approach to entropic approximations of optimal transport problems

Luigi De Pascale

Abstract: in this talk I will present a minimalist approach to the entropic approximations of optimal transport problems. This approach seems to allow some small generalisation, in the direction of requiring less regularity of the pointwise transport cost, From a paper in preparation with Camilla Brizzi and Anna Kausamo.

Riesz inequality for polygons: symmetry and symmetry breaking.

Ilaria Fragalà

Abstract: I will discuss counterparts of the classical Hardy-Littlewood and Riesz inequalities when the class of admissible domains is the family of polygons with a fixed number of sides. The latter corresponds to study the polygonal isoperimetric inequality in nonlocal version. Based on a joint work with Beniamin Bogosel and Dorin Bucur.

Controllo ottimo di un'epidemia: meglio attendere o intervenire subito?

Lorenzo Freddi

Abstract: Le strategie di controllo non farmaceutico di un'epidemia possono essere di tipo farmaceutico (e.g., vaccini o farmaci antivirali) oppure non farmaceutico (ad esempio distanziamento, isolamento, quarantena, dispositivi di protezione personale, lockdown). Matematicamente, l'epidemia può essere modellata come sistema dinamico. Il problema di come controllarla in maniera ottimale, in modo da minimizzare i costi di controllo, sociali, economici e sanitari, è formulabile come "problema di controllo ottimo". Nel seminario verranno presentati, con strumenti elementari, alcuni recenti risultati di controllabilità e sintesi di strategie di controllo ottimo per un'epidemia SIR nel caso in cui si voglia mantenere il numero di infetti al di sotto di una prestabilita soglia di sicurezza atta a garantire il buon funzionamento del sistema sanitario. Questo tipo di problema si è presentato in maniera naturale durante la recente pandemia di Covid-19 quando limitare il numero di infetti serviva ad evitare di intasare i reparti di terapia intensiva. Il fatto che il vincolo sia imposto su una funzione di stato colloca quello in esame in una classe di problemi di controllo ottimo con vincolo di stato particolarmente studiata nell'ultimo decennio con strumenti di analisi che vanno dalle funzioni a variazione limitata alle disequazioni differenziali. Tuttavia, oltre alla formulazione, anche la deduzione di gran parte dei risultati ottenuti, come lo studio della controllabilità e la sintesi di strategie ottimali, può essere affrontata con strumenti elementari.

Optimal Transport methods in Electronic Structure Theory

Augusto Gerolin

Abstract: In this presentation, I will revisit the pioneering work of G. Buttazzo, L. De Pascale, and P. Gori-Giorgi, which establishes a connection between Optimal Transport Theory for Coulomb costs and the Strictly Correlated Electron Limit of Density Functional Theory. Additionally, I will highlight recent advancements in the field.

Manipulation of social choice functions under incomplete information

Michele Gori

Abstract: An interesting and recent problem in social choice theory is the analysis of how vulnerable to manipulation social choice functions are when a limited amount of information about individual preferences is available. We focus then on a property called WMG-strategy-proofness. A social choice function is WMG-strategy-proof if it cannot be manipulated by an individual whose information about the preferences of the others is limited to the knowledge, for every pair of alternatives, of the number of people preferring the first alternative to the second one. We prove that, if the alternatives are at least three, there are Pareto optimal, WMG-strategy-proof and non-dictatorial social choice functions and that every Pareto optimal and WMG-strategy-proof social choice function is not anonymous.

Magia Matematica

Luca Granieri

Abstract: Formule e giochi di prestigio, calcoli e divinazioni, dimostrazioni e trucchi. Magia e Matematica. Alle radici della scienza moderna c'è la matematica intrecciata al pensiero magico. La scienza moderna può essere interpretata come una sorta di magia matematizzata, indispensabile per vivere l'avventura della scoperta scientifica e la bellezza del sapere, e sprigionare la meraviglia di un modo tutto nuovo di conoscere. Il metodo scientifico e il pensiero magico si compenetrano anche per per offrire a insegnanti, alunni e appassionati un'esperienza da condividere e raccontare.

BMO-type functionals, total variation and denoising models

Serena Guarino Lo Bianco

Abstract: We present some results concerning the approximation of the total variation of a function by a family of BMO-type functionals. The mode of convergence is rather delicate as for a general BV function the pointwise limit does not always exist. De Giorgi's concept of Γ-convergence save the situation. We also deal with a minimization problem coming from applications in image processing. Based on joint works with R. Schiattarella.

Optimal configurations of decorated phases for a model for block copolymers

Xin Yang Lu

Abstract: Energies governing the behavior of block copolymers are often made of a local term, e.g. perimeter, plus a long range interaction, e.g. Coulomb-like repulsion. Generally speaking, the former prefers fewer but bigger components, while the latter prefers smaller but more numerous components. Therefore, optimal configurations must strike a balance between those two competing forces. In this talk, we will present some recent results on the geometry of optimal configurations.

Gripenberg's Algorithm for the Lower Spectral Radius

Francesco Paolo Maiale

Abstract: In this talk, we discuss recent developments in computing the spectral radii of matrix families, focusing on the lower spectral radius (LSR) and the joint spectral radius (JSR). These quantities characterize, respectively, the lowest and fastest growth rate of sequences in the product semigroup generated by a matrix family $\mathcal{F} = \{A_1, \dots, A_m\}$. The LSR, in particular, plays a crucial role in identifying optimal stable trajectories in discrete dynamical systems. However, its computation has been a challenge due to the lack of readily available lower bounds.

We introduce an extension of Gripenberg's algorithm (1996), specifically designed to compute the LSR for families of non-negative matrices. Our approach adaptively refines a polytope antinorm to bound the LSR and guarantees convergence within a pre-selected precision $\delta > 0$. We also discuss the theoretical results behind this algorithm, which rely on the recent finding that the LSR can be obtained as a Gelfand-type limit (Guglielmi and Zennaro, 2020).

Next, we compare the performance of our adaptive algorithm to the polytopic algorithm (Pratasov and Guglielmi, 2013) in computing the LSR. This comparison will show the advantages and disadvantages of each method, focusing on randomly generated matrix families with low sparsity density and specific applications in number theory (e.g., Euler binary partition functions).

Finally, we discuss a refinement to the classical Gripenberg's algorithm for approximating the JSR of arbitrary matrix families. The same adaptive procedure used for the LSR can be implemented to adaptively refine polytope norms, leading to faster convergence of the upper bound to the JSR.

The talk is based on joint work with Nicola Guglielmi (Gran Sasso Science Institute).

On the sharp Makai inequality

Francesca Prinari

Abstract: In this talk we present a generalisation of a result shown by E. Makai for the torsional rigidity of a planar convex bounded open set. More precisely, we establish that on every N-dimensional convex open set, Poincaré–Sobolev constants for functions vanishing at the boundary can be bounded from below in terms of the norm of the distance function in a suitable Lebesgue space. In addition, we compare the sharp Makai constants obtained in the class of convex sets with the optimal constants defined in other classes of open sets. Based on joint works with A.C. Zagati and Lorenzo Brasco.

An open problem around the Kohler-Jobin inequality

Berardo Ruffini

Abstract: We discuss an open problem about some limiting cases of the Kohler-Jobin inequality and some strategy to solve it (or not).

Dalla pianificazione urbana allo schema JKO

Filippo Santambrogio

Abstract: Il punto di partenza di questo intervento sarà il mio primo lavoro con Giuseppe, che risale ad una ventina d'anni fa. Un articolo che di per sé non è granché (ma SIAM ce l'ha addirittura ripubblicato come SIGEST in SIAM Review), dedicato ad un modello di pianificazione urbana (dove mettere in modo ottimale servizi e abitanti). L'obiettivo del seminario è di mostrare che le idee discusse con Giuseppe a quel tempo sono sempre presenti nella mia ricerca, fino agli argomenti su cui lavoro ora attorno alla discretizzazione temporale di equazioni di diffusione (schema JKO).

Current vortex sheets in ideal compressible MHD

Paola Trebeschi

Abstract: In this talk we are concerned with nonlinear stability and existence of two-dimensional current-vortex sheets in ideal compressible magnetohydrodynamics. This is a nonlinear hyperbolic initial-boundary value problem with characteristic free boundary. It is well-known that current-vortex sheets may be at most weakly (neutrally) stable due to the existence of surface waves solutions that yield a loss of derivatives in the energy estimate of the solution with respect to the source terms. We first identify a sufficient condition ensuring the weak stability of the linearized current-vortex sheets problem. Under this stability condition for the background state, we show that the linearized problem obeys an energy estimate in anisotropic weighted Sobolev spaces with a loss of derivatives. Based on the weakly linear stability results, we then establish the local-in-time existence and nonlinear stability of current-vortex sheets by a suitable Nash-Moser iteration, provided the stability condition is satisfied at each point of the initial discontinuity. This result gives a new confirmation of the stabilizing effect of sufficiently strong magnetic fields on Kelvin-Helmholtz instabilities.

This is a joint work with A. Morando (Brescia), P.Secchi (Brescia) and D. Yuan (Beijing Normal Univ.)

Optimal partition problems

Bozhidar Velichkov

Abstract: We present a recent joint work with Roberto Ognibene on the problem of optimal partition of a fixed domain (a box) with respect to the sum of the principal eigenvalues and we prove regularity results for the free interface up to fixed boundary; in particular, we prove that the subset of points of minimal frequency is regular and that the interior free interface is approaching the boundary orthogonally in a smooth way.
