Booklet of abstracts and timetable

Workshop

Regularity and geometric aspects of nonlinear PDEs

Pisa, 31 Jan. – 2 Feb. 2024

This workshop is organized with the support of the following projects:

- MUR-PRIN project 2022R537CS "NO^3 – Nodal optimization, nonlinear elliptic equations, nonlocal geometric problems, with a focus on regularity", granted by the European Union – Next Generation EU;
- ERC project "VAREG – Variational approach to the regularity of the free boundaries" (grant argeement n.853404), granted by the European Research Council (ERC) within the European Union's Horizon 2020 research and innovation programme.

TIMETABLE

Regularity and geometric aspects of nonlinear PDEs

31 Jan – 2 Feb 2024

The workshop will take place in

Aula Magna of Dipartimento di Matematica, Università di Pisa

Wednesday 31st
14:00–14:30 Registration
14:30–15:30 Cristiana De Filippis
15:30–16:30 Daniele Valtorta
16:30–17:00 Coffee break
17:00–18:00 Stefano Vita
18:00–19:00 Roberto Ognibene

Thursday 1st
09:30–10:30 Chiara Saffirio
10:30–11:00 Coffee Break
11:00–12:00 Hugo Tavares
12:00–13:00 Idriss Mazari-Fouquer
15:00–16:00 Matteo Muratori
16:00–16:30 Coffee break
16:30–17:30 Stefano Biagi
17:30–18:30 Azahara De La Torre
18:30–19:30 Discussion

Friday 2nd
09:30–10:30 Isabella Ianni
10:30–11:00 Coffee Break
11:00–12:00 Nicola Abatangelo
12:00–13:00 Denis Bonheure
13:00–14:00 Discussion and closure

ABSTRACTS

Regularity and geometric aspects of nonlinear PDEs

31 Jan – 2 Feb 2024

An elliptic boundary value problem with fractional nonlinearity

Nicola Abatangelo
Università di Bologna

Abstract:
We present results concerning the existence and uniqueness of solutions to a nonlinear equation driven by an operator arising from the superposition of a Laplacian and a signed power of a fractional Laplacian, which is a problem with hidden unavoidable singularities to be taken care of. The main focus is on the boundary conditions naturally associated to the equation. We also go over a couple of related problems. This is a joint work with M. Cozzi (Milan).

A Brezis-Nirenberg type result for mixed local and nonlocal operators

Stefano Biagi
Politecnico di Milano

Abstract: In this talk we present some existence results, in the spirit of the celebrated paper by Brezis and Nirenberg (CPAM, 1983), for a perturbed critical problem driven by a mixed local and nonlocal linear operator. More precisely, we develop an existence theory in the cases of linear, superlinear and singular perturbations; in the particular case of linear perturbations, we also investigate an associated mixed Sobolev inequality, detecting the optimal constant, which we show that is never achieved. The results discussed in this seminar are obtained in collaboration with S. Dipierro, E. Valdinoci and E. Vecchi.

An overview on the Born-Infeld equation

Denis Bonheure
Université Libre de Bruxelles

Abstract: I will give an overview of the recent advances made concerning the understanding of the electro-static Born-Infeld energy functional. The associated PDE – usually called Born-Infeld Equation - is a quasilinear (and singular) elliptic equation with source. This fascinating equation can be regarded as the mean curvature equation in Lorentz-Minkowski space. Special attention will be given to unsolved issues.

Nonlinear potentials at the fractional scale: sharp regularity

Cristiana De Filippis
Università di Parma

Abstract: Nonlinear potential theory and elliptic regularity theory are two classical topics in the modern analysis of partial differential equations. In this talk I show how these themes merge to solve the longstanding open problem, dating back to the seminal contributions of Ladyzhenskaya & Ural'tseva, Trudinger, and Simon (1967-1976), of deriving Schauder estimates for minima of functionals (resp. solutions to elliptic equations) featuring polynomial nonuniform ellipticity. The sharp rate of nonuniform ellipticity for the validity of Schauder theory is also disclosed. From recent, joint work with Giuseppe Mingione (Parma).

Uniqueness of least-energy solutions to the fractional Lane-Emden equation in the ball.

Azahara De La Torre
Sapienza Università di Roma

Abstract: In this talk we will show uniqueness of least-energy solutions for the fractional Lane-Emden equation posed in the ball under homogeneous Dirichlet exterior conditions. This is a non-local semilinear equation with a superlinear and subcritical nonlinearity. Existence of positive solutions follows easily from variational methods, but uniqueness is quite complicated. In the local case, uniqueness of positive solutions follows from the result of Gidas, Ni and Nirenberg. Indeed, by using the moving plane method, they proved radial symmetry of the solutions which allows to apply ODE techniques. In the non-local case these arguments don't seem to work. Our proof makes use of Morse theory, and it is inspired by some results obtained by C. S. Lin in the '90s. The talk is based in a joint work with Enea Parini.

Uniqueness and nondegeneracy for fractional Dirichlet problems

Isabella Ianni
Sapienza Università di Roma

Abstract: We discuss some recent uniqueness and nondegeneracy results for non-negative solutions of some fractional semilinear problems in bounded domains with Dirichlet exterior condition. In particular we can consider least energy solutions in balls or in more general symmetric domains, for problems with power nonlinearities. The symmetry properties of the solutions of the associated linearized equation are also investigated. The talk is mainly based on the following joint works:

[1] A. Dieb, I. Ianni, A. Saldaña, Uniqueness and nondegeneracy for Dirichlet fractional problems in bounded domains via asymptotic methods, Nonlinear Analysis, 236, 2023.
[2] A. Dieb, I. Ianni, A. Saldaña, Uniqueness and nondegeneracy of least-energy solutions to fractional Dirichlet problems, preprint arXiv:2310.01214

The Faber-Krahn inequality for the Stokes operator

Idriss Mazari-Fouquer
Université de Paris Dauphine

Abstract: In this work in collaboration with D. Bucur, A. Henrot and Y. Privat, we investigate the minimisation of the first eigenvalue of the Dirichlet-Stokes operator in $\mathbb{R}^d$. To be more specific, we consider the question of existence of optimal shapes, a query made more difficult by the incompressibility constraint, as well as the minimality of the ball. This latter point turns out to be dependent on the dimension, and starkly different from the case of the (scalar) Dirichlet-Laplacian.

Rigidity results for the Lane-Emden equation on Cartan-Hadamard manifolds

Matteo Muratori
Politecnico di Milano

Abstract: The Cartan-Hadamard conjecture states that on an arbitrary n-dimensional Cartan-Hadamard manifold $\mathbb{M}^n$ the isoperimetric inequality holds with *Euclidean* optimal constant, and equality holds for a bounded Borel set E if and only if E is isometric to a Euclidean ball (up to a negligible set). So far, the conjecture has been proved only in dimension $n \leq 4$. Moreover, it is known that its validity implies in turn the validity of the *p-Sobolev inequality* for all $p \in (1, n)$, still with Euclidean optimal constant.

In the first part of the talk, I will discuss some recent results regarding the classification of Cartan-Hadamard manifolds that admit the existence of optimal functions for the p-Sobolev inequality. Specifically, we show that, under the validity of the above mentioned Cartan-Hadamard conjecture, the only Cartan-Hadamard manifold for which such optimal functions exist is the Euclidean space $\mathbb{R}^n$ (up to isometries).

In the second part of the talk, I will focus on the classification of *radial solutions* of the corresponding Euler-Lagrange equation, which need not be optimizers of the p-Sobolev inequality; the latter turns out to be a semilinear elliptic equation of *Lane-Emden* type. More generally, we study the *critical* or *supercritical* equation

$$-\Delta_p u = u^q \quad \text{on } \mathbb{M}^n\,, \qquad u > 0\,,$$

where $q \geq p^* - 1$ and $p^* = \frac{pn}{n-p}$. We prove that, if $\mathbb{M}^n$ is spherically symmetric (i.e. a Cartan-Hadamard *model manifold*) and there exists a radial solution of such equation with *finite energy*, then $\mathbb{M}^n$ is necessarily isometric to $\mathbb{R}^n$ and u is an Aubin-Talenti function. Furthermore, we show that solutions with infinite energy do exist, and provide a detailed description of their asymptotic behavior depending on the *p-stochastic completeness* or *incompleteness* of $\mathbb{M}^n$. A similar analysis, in the case $p = 2$, is also performed for the Lane-Emden *system*, where several technical difficulties arise.

The talk is based on joint works with Nicola Soave.

Free boundary regularity in an optimal partition problem

Roberto Ognibene
Università di Pisa

Abstract: Let us consider a bounded domain, divided into a fixed number of disjoint subdomains and, among all the possible configurations, let us consider the one for which the sum of the first Dirichlet eigenvalues of the subdomains is minimal. In this talk, I will discuss the regularity of the interface which emerges as boundary of such optimal partition and, in particular, I will focus on the regularity up to the fixed boundary. The talk is based on a joint work with B. Velichkov.

Uniqueness criteria for the Vlasov-Poisson system and applications to semiclassical problems.

Chiara Saffirio
Basel Universität

Abstract: The Vlasov-Poisson system is a non-linear kinetic equation describing the mean-field time-evolution of particles forming a plasma. In this talk I will present uniqueness criteria for Vlasov-Poisson type equations, emerging as corollaries of stability estimates in strong topologies (associated with Lebesgue norms) or in weak topologies (induced by Wasserstein distances), and show how they serve as a guideline to study the classical limit from the Hartree equation to the Vlasov equation in different settings.

Classification and stability of positive solutions to the NLS equation on some particular metric graph

Hugo Tavares
Instituto Superior Técnico Lisboa

Abstract: Given $\lambda > 0$ and $p > 2$, we present a complete classification of the positive H^1-solutions of the equation $-u'' + \lambda u = |u|^{p-2}u$ on the $\mathcal{T}$-metric graph (consisting of two unbounded edges and a terminal edge of length $\ell > 0$, all joined together at a single vertex). This study implies, in particular, the uniqueness of action ground states. Moreover, for $p \sim 6^-$, the notions of action and energy ground states do not coincide and energy ground states are not unique. In the L^2-supercritical case $p > 6$, we prove that, for $\lambda \sim 0^+$ and $\lambda \sim +\infty$, action ground states are orbitally unstable for the flow generated by the associated time-dependent NLS equation $i\partial_t u + \partial_{xx}^2 u + |u|^{p-2}u = 0$ (providing numerical evidence of the existence of both stable and unstable action ground states for $p \sim 6$). Finally, we discuss related results for the tadpole and other particular metric graphs. This is a joint work with Francisco Agostinho and Simão Correia.

Energy Identity for Stationary Harmonic Maps

Daniele Valtorta
Università di Milano Bicocca

Abstract: We present the proof for Energy Identity for stationary harmonic maps. In particular, given a sequence of stationary harmonic maps weakly converging to a limit with a defect measure for the energy, then m-2 almost everywhere on the support of this measure the density is the sum of energy of bubbles. This is equivalent to saying that annular regions (or neck regions) do not contribute to the energy of the limit.

This result is obtained via a quantitative analysis of the energy in annular regions for a fixed stationary harmonic map. The proof is technically involved, but it will be presented in simplified cases to try and convey the main ideas behind it.

Degenerate equations on nodal sets

Stefano Vita
Università di Torino

Abstract: We discuss some regularity results for a class of elliptic equations with coefficients that degenerate or explode on the nodal set $Z(u)$ of a given function u. This function is a solution to a uniformly elliptic PDE. The analysis is strongly influenced by the singular set of u. Some uniformity of the estimates is provided under bounds on the frequency function. This is a joint project with Susanna Terracini and Giorgio Tortone.
