Blow-up limits and uniform concentration property for Griffith minimizers

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The Griffith functional was introduced by Francfort and Marigo to model the equilibrium state of a fracture in linear elasticity. Let Ω be a bounded open subset of $\mathbf{R}^N$, which represents the reference configuration (without fracture) of a solid. We apply a deformation at the boundary of Ω and we assume that the material only undergoes elastic deformations before breaking (brittle material). Francfort and Marigo define the equilibrium state as a minimizer of the functional

$$\mathcal{G}(u,K) := \int_{\Omega \setminus K} |e(u)|^2 \, dx + \mathcal{H}^{N-1}(K),$$

among pairs (u,K) such that K is a subset of dimension $(N-1)$ of Ω (the fracture), $u\colon \Omega \setminus K \to \mathbf{R}^N$ is a C^1 function (a displacement field) which satisfies a Dirichlet condition at the boundary $\partial\Omega$ and the matrix $e(u) := (Du + Du^T)/2$ is the symmetric part of the gradient of u.

The study of blow-up limits is at the heart of the local description of fracture. The goal of this talk is to present a recent work in collaboration with A. Lemenant, where we prove that blow-up limits in Hausdorff distance are global minimizers and we classify the homogeneous global minimizers in dimension 2. For this purpose, we developed a new approach to the uniform concentration property of Dal Maso, Morel and Solimini in the vectorial case.